Oil Spill Response Planning with MINLP

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1. Introduction

Catastrophic oil spills [1], such as the recent Deepwater BP oil spill in the Gulf of Mexico [2], have demonstrated the importance of developing responsive and effective oil spill response planning strategies for the oil industry and the government. Although a few models have been developed for oil-spill response planning, response operations and the oil weathering process are usually considered separately [3, 5, 4]. Yet significant interactions between them exist throughout the response. Oil-spill cleanup activities change the volume and area of the oil slick and in turn affect the oil transport and weathering process, which also affects coastal protection activities and cleanup operations (e.g., performance degradation and operational window of cleanup facilities). Therefore, it is critical to integrate the response planning model with the oil transport and weathering model, although this integration has not been addressed in the existing literature to the best of our knowledge.

The objective of this note is to develop an optimization approach for seamlessly integrating the planning of oil-spill response operations with the oil transport and weathering process. A mixed-integer dynamic optimization (MIDO) model is proposed that simultaneously predicts the time trajectories of the oil volume and slick area and the optimal response cleanup schedule and coastal protection plan, by taking into account the time-dependent oil physiochemical properties, spilled amount, hydrodynamics, weather conditions, facility availability, performance degradation, cleanup operational window, and regulatory constraints. To solve the MIDO problem, we reformulated it as a mixed-integer nonlinear programming (MINLP) problem using orthogonal collocation on finite elements. We also developed a mixed-integer linear programming (MILP) model to obtain a good starting point for solving the nonconvex MINLP problem. The application of the proposed integrated optimization approach is illustrated through a case study based on the Deepwater BP oil spill.

The rest of this paper is organized as follows. Section 2 presents the problem statement. The detailed model formulation is given in Section 3. In Section 4, we present computational results for the case study.

2. Problem Statement

The problem addressed in this work can be formally stated as follows. An oil spill occurs at a specific location. The initial spill volume, constant release rate, and release duration are all known. We are also given the physical and chemical parameters of the oil and seawater, as well as the weather data, such as wind speed and temperature. There is a set of staging areas $i \in I$ along the shoreline near the spill site. Because of spreading and drift processes, the oil slick may hit the coast around staging area i at time t if the slick area is larger than $\overline{AREA}_{i,t}$, which is a given parameter in this work and can be derived from the drift process based on weather and sea conditions. A minimum length of boom $\underline{L}_i$ should be deployed in staging area i before the oil slick hits the corresponding coast. The maximum boom deployment rates and the unit boom deployment cost are given. Booms deployed around staging area i will be subject to failure after a lifetime φ_i. The major cleanup methods include mechanical cleanup and recovery (skimming), in situ burning, and chemical dispersant application; and the corresponding cleanup facilities are indexed by m, b, and d, respectively. The maximum number of each type of cleanup facility in each staging area and the corresponding total response time are known. The operating capacities of the cleanup facilities and the corresponding operational costs and operating conditions, as well as the time-dependent weather factors, are given. When the response operations finish, the volume of oil remaining on the sea surface should not exceed the cleanup target $\underline{V}$. The problem is to simultaneously determine the coastal protection plan and cleanup schedule in order to minimize the total response cost under a specific response time span.

3. Mixed-Integer Dynamic Optimization Model

The proposed MIDO model has an objective function to minimize the total response cost, and includes a set of ordinary differential equations (ODEs) for the oil transport and weathering process and a set of mixed-integer constraints for response planning. The oil weathering model uses a continuous-time representation, while in the planning model we discretize the planning horizon into $|T|$ time periods with H_t as the length of time period $t \in T$. This is consistent with the real-world practice that most response decisions are made on an hourly or daily basis. The integration of these two time representations will be discussed at the end of this section.

3.1 Oil transport and weathering model

Prediction of the oil transport and weathering process needs to account for many factors, such as oil properties, spilled amount, hydrodynamics, and weather conditions, and to consider a variety of complex physicochemical processes taking place simultaneously. Over 50 oil weathering models, based on empirical and semi-empirical approaches, have been developed. Although any oil weathering model can, in principle, be used in the approach proposed in this paper, we employ a dynamic mathematical model taking into account the dominant processes (spreading, evaporation, emulsification, and dispersion) that cause significant short-term changes in oil characteristics. We note that a PDE model might better capture the physiochemical evolution of oil slick in the three dimension space, it might be challenging to be integrated with the response planning model. Thus, we consider only the time variation in area, volume and other important physiochemical parameters, and model the effect of wind and current through parameters with constant values.

Spreading, which strongly influences coastal protection operations and other weathering processes, is probably the most dominant process of a spill. The rate of change of slick area is given by [8, 9, 10]:

$$\frac{dA_{(t)}}{dt} = K_1 A_{(t)}^{-1} V_{(t)}^{3/4} - W_{(t)} \frac{A_{(t)}}{V_{(t)}} \tag{1}$$

where A is the surface area of oil slick (m^2), V is the volume of oil (m^3), and K_1 is the physicochemical parameters of the crude oil with a default value, and $W_{(t)}$ is the cleanup rate given in Equation (26). The first term on the right-hand side of Equation (1) is for the natural spreading process. The second term refers to the reduction of slick area as a result of cleanup operations. Symbols with subscript (t) are time-dependent variables. The initial area of oil slick can be determined by the well-known gravity-viscous formulation: [11]

$$A_0 = \pi \cdot \frac{k_2^4}{k_3^2} \Big[\frac{(\rho_w - \rho_o) V_0^5 g}{\rho_w \nu_w} \Big]^{1/6} \tag{2}$$

where g is the acceleration of gravity, ρ_w is seawater density, ρ_o is the density of fresh oil, nu_w is the kinematic viscosity of seawater, V_0 is the initial volume, and k_2 and k_3 are empirical constants.

The volume balance of the oil slick is based on the volume variation rate given as follows: [12, 10]

$$\frac{dV_{(t)}}{dt} = -V_{(t)} \frac{dF_{E(t)}}{dt} - \frac{dV_{D(t)}}{dt} - W_{(t)} + VI_{(t)} \tag{3}$$

where the first term on the right-hand side is for the evaporation loss, the second term is for natural dispersion, the third term is the cleanup rate, and the last term is the oil spill rate given as follows:

$$VI_{(t)} = \begin{cases} \text{constant spill rate,} & 0 \le t \le tf_1 \\ 0, & tf_1 \le t \le tf_2 \end{cases} \tag{4}$$

where tf_1 is the time when the oil spillage stops and tf_2 is the final time of the planning horizon (response time span). The initial volume of the oil slick is given as V_0, and the remaining volume of oil at the end of the response time span should not exceed the cleanup target.

$$V_{(t=0)} = V_0, \quad V_{(t=tf_2)} \le \underline{V} \tag{5}$$

Evaporation is the primary initial process involved in the removal of oil from sea. The rate that oil evaporates from the sea surface is modeled by the following equation: [13]

$$\frac{dF_{E(t)}}{dt} = \frac{K_{ev} A_{(t)}}{V_{(t)}} \cdot \exp\Big[A_{ev} - \frac{B_{ev}}{T_K} \big(T_0 + T_G F_{E(t)} \big) \Big] \tag{6}$$

where F_E is the volume fraction of oil that has been evaporated, T_K is the oil temperature (K), which is assumed to be a constant, K_{ev} is the mass transfer coefficient for evaporation, T_0 is the initial boiling point, T_G is the gradient of the oil distillation curve, and A_{ev} and B_{ev} are empirical constants. As no oil was evaporated at time 0, the initial value of evaporative fraction is given by

$$F_{E(t=0)} = 0 \tag{7}$$

The rate of dispersion into the water column of the floating oil slick is given by the following: [8, 9, 12]

$$\frac{dV_{D(t)}}{dt} = \frac{0.11 \cdot (WS+1)^2 \cdot A_{(t)} V_{(t)}}{A_{(t)} + 50\zeta \cdot V_{(t)} \mu_{(t)}^{1/2}} \tag{8}$$

where V_D is volume of oil naturally dispersed, WS is the wind speed, and ζ is the oil-water interfacial tension. The initial value of the volume of oil that is naturally dispersed is zero.

$$V_{D(t=0)} = 0 \tag{9}$$

In emulsification, water droplets are entrained in the oil. The dynamic emulsification process that incorporates water into oil can be computed with the following equation: [8]

$$Y_{W(t)} = C_3 \Big[1 - \exp\big(- \frac{K_{em}}{C_3} (WS+1)^2 \cdot t \big) \Big] \tag{10}$$

where Y_W is the fractional water content in the emulsion, C_3 is a viscosity constant for the final fractional water content, K_{em} is an empirical constant and t is the time in seconds.

As a result of both Mousse formation and evaporation, the viscosity of oil slick may significantly increase during the emulsification process. The rate of changes in viscosity is given by: [14, 12]

$$\frac{d\mu_{(t)}}{dt} = \frac{2.5\mu_{(t)}}{\big(1 - C_3 Y_{W(t)}\big)^2} \frac{dY_{W(t)}}{dt} + C_4 \mu \frac{dF_{E(t)}}{dt} \tag{11}$$

where μ is the viscosity of the oil and C_4 is an oil-dependent constant. The initial value of the viscosity is the same as the parent oil's viscosity, which can be calculated by the following equation: [14]

$$\mu_0 = 224 \times \sqrt{AC} \tag{12}$$

where AC is the asphaltene content of the parent oil.

3.2 Response planning constraints

We consider both coastal protection and oil-spill cleanup operations in the response. The major coastal protection method is to deploy booms to prevent the oil from spreading to the shore. Three major oil spill cleanup methods are mechanical cleanup and recovery, in situ burning, and chemical dispersants. Mechanical systems can skim the oil slick and recover oil from the emulsion; in situ burning and chemical dispersants remove oil only from the surface of the sea. Reviews of oil spill response methods and equipment are given in [15, 10].

In order to protect sensitive shorelines, either the slick area must be controlled through effective cleanup operations, or coastal protection booms must be deployed with sufficient lengths around the staging areas before they are hit by the oil slick. It can be modeled with the following constraint:

$$A_{(t)} \le \overline{AREA}_{i,t} + A^U \cdot z_{i,t}, \quad \forall i \in I, t \in T \tag{13}$$

where $z_{i,t}$ is a binary variable that equals 1 if sufficient booms have been deployed to protect the shoreline around staging area i at time t, A^U is the upper bound of oil slick area, and $\overline{AREA}_{i,t}$ is a given parameter for the area of the oil slick that will hit the shore around staging area i at time period t. $\overline{AREA}_{i,t}$ depends primarily on the drift process, which relates to the wind speed and direction. [10]

The shoreline around staging area i is fully protected by the booms at time period t if and only if the length of boom is no less than the required length. So we have:

$$\underline{L}_i \cdot z_{i,t} \le bl_{i,t} \le \underline{L}_i + L^U \cdot z_{i,t}, \quad \forall i \in I, t \in T \tag{14}$$

where $\underline{L}_i$ is the length of boom required to protect the coast around staging area i and $bl_{i,t}$ is the length of boom deployed along the shore of staging area i at the end of time period t.

Because of currents and winds, conventional booms are subject to damages over time. Coastal protection booms deployed at staging area i can be effective for only a certain lifetime φ_i after deployment. Booms deployed at time $t - \varphi_i$ will fail at time t. [7] Thus, the length of the boom around the shore of staging area i at the end of time period t ($bl_{i,t}$) is equal to the boom length at the end of the previous time period ($bl_{i,t-1}$) plus the length of the boom deployed at the current time period ($bdep_{i,t}$) minus those that fail at this time period ($bdep_{i,t-\varphi_i}$). Thus, the balance of boom length is given as follows.

$$bl_{i,t} = bl_{i,t-1} + bdep_{i,t} - bdep_{i,t-\varphi_i}, \quad \forall i \in I, t \in T \tag{15}$$

The length of boom deployed along the shoreline near staging area i at time t should not exceed the maximum deployment rates ($BDU_{i,t}$) times the length of time period t (H_t). Thus, we have:

$$0 \le bdep_{i,t} \le BDU_{i,t} \cdot H_t, \quad \forall i \in I, t \in T \tag{16}$$

We define $x^M_{i,m,t}$ as the number of mechanical cleanup and recovery systems type m from staging area i operating at time period t. It should not exceed the corresponding available number ($N^M_{i,m,t}$).

$$0 \le x^M_{i,m,t} \le N^M_{i,m,t}, \quad \forall i \in I, m \in M, t \in T \tag{17}$$

The volume of oil cleaned and recovered from the sea surface with mechanical systems at time period t (W^M_t) is given by the following equation. [5]

$$W^M_t = \sum_i \sum_m (1 - Y_{W(t)}) \cdot H_t \cdot \omega^M_t \cdot Q^M_{i,m} \cdot x^M_{i,m,t}, \forall t \in T \tag{18}$$

where $Q^M_{i,m}$ is the operating capability of mechanical system m from staging area i; ω^M_t is the weather factor (between 0 and 1), which can be determined from weather forecasting; and $Y_{W(t)}$ is the fractional water content defined in (10).

In situ burning response system b can operate only when the oil slick (δ_t) is thicker than δ^B_b. [6] We introduce a binary variable ($xx^B_{b,t}$) to model this restriction through the following constraint:

$$\delta^B_b \cdot xx^B_{b,t} \le \delta_{(t)} \le \delta^B_b + \delta^U \cdot xx^B_{b,t} \tag{19}$$

where δ^U is the upper bound of the slick thickness and $\delta_{(t)}$ is the thickness of the oil slick given by:

$$\delta_{(t)} \cdot A_{(t)} = V_{(t)} \tag{20}$$

For in situ burning response systems, the availability constraints are given as follows:

$$x^B_{i,b,t} \le N^B_{i,b,t} \cdot xx^B_{b,t}, \quad \forall i \in I, b \in B, t \in T \tag{21}$$

where $x^B_{i,b,t}$ is the number of in situ burning systems b from staging area i operating at time period t, and $N^B_{i,b,t}$ is the availability in staging area i.

The volume of oil burned at time period t (W^B_t) is given by the following equation:

$$W^B_t = \sum_i \sum_b H_t \cdot \omega^B_t \cdot Q^B_{i,b} \cdot x^B_{i,b,t}, \quad \forall t \in T \tag{22}$$

where $Q^B_{i,b}$ is the operating capability of in situ burning system b from staging area i and ω^B_t is the weather factor for in situ burning at time t.

The availability constraint of chemical dispersant application systems is given by:

$$x^D_{i,d,t} \le N^D_{i,d,t} \cdot \gamma_{i,d,t} \cdot H_t, \forall i \in I, d \in D, t \in T \tag{23}$$

where $x^D_{i,d,t}$ is the number of sorties of chemical dispersant application systems d dispatched from staging area i at time period t, $N^D_{i,d,t}$ is the corresponding availability, and $\gamma_{i,d,t}$ is the maximum number of sorties of dispersant application systems d from staging area i to spray dispersant on the oil slick at time period t. Note that the maximum number of sorties depends on the type of dispersant application system (e.g., a helicopter may operate 10 sorties per day for an offshore oil spill 100 miles away). [16]

The volume of oil removed from the sea surface by using chemical dispersants at time period t (W^D_t) is given by the following equation:

$$W^D_t = \sum_i \sum_d \omega^D_t \cdot \rho^{effect}_t \cdot \rho^{accuracy}_d \cdot Q^D_{i,d} \cdot x^D_{i,d,t}, \quad \forall t \in T \tag{24}$$

where $Q^D_{i,d}$ is the operating capacity of dispersant application systems d from staging area i, ω^D_t is the corresponding weather factor, ρ^{effect}_t is the effectiveness factor (ratio between oil dispersed and dispersant sprayed) for chemical dispersant application at time t, and $\rho^{accuracy}_d$ is the accuracy factor (percentage of sprayed dispersant falls on the oil slick) of dispersant application systems d.

The total amount of chemical dispersant used throughout the entire response operation should not exceed the limit set by the regulator ($DLIMIT$). [16]

$$\sum_i \sum_d Q^D_{i,d} \cdot x^D_{i,d,t} \le DLIMIT \tag{25}$$

We model the real-time cleanup rate ($W_{(t)}$) as a piece-wise step function as follows:

$$W_{(t)} \cdot H_t = W_t^M + W_t^B + W_t^D \tag{26}$$

The decisions variables of this model should satisfy the integrality and/or nonnegativity constraints given as follows:

$$Z_{i,t} \in \{0,1\} \tag{27}$$
$$x_{i,m,t}^M, x_{i,b,t}^B, x_{i,d,t}^D \in \{0\} \cup Z^+ \tag{28}$$
$$bdep_{i,t}, bl_{i,t}, W_t^M, W_t^B, W_t^D \geq 0 \tag{29}$$
$$V_{(t)}, A_{(t)}, F_{E(t)}, V_{D(t)}, Y_{W(t)}, \mu_{(t)}, \delta_{(t)} \geq 0 \tag{30}$$

3.3 Objective functions

The objective function is to minimize the total response cost, given as follows.

$$\begin{aligned} \min: \quad TotalCost &= \sum_i \sum_m \sum_t C_{i,m,t}^M \cdot x_{i,m,t}^M \\ &+ \sum_i \sum_b \sum_t C_{i,b,t}^B \cdot x_{i,b,t}^B \\ &+ \sum_i \sum_d \sum_t C_{i,d,t}^D \cdot x_{i,d,t}^D \\ &+ \sum_i \sum_t CDEP_{i,t}^{boom} \cdot bdep_{i,t} \\ &- \sum_t OC \cdot W_t^M \end{aligned} \tag{31}$$

where $C_{i,m,t}^M$, $C_{i,b,t}^B$ and $x_{i,d,t}^D$ are unit operating cost of mechanical cleanup and recovery system, in situ burning system and chemical dispersant application system, respectively, $CDEP_{i,t}^{boom}$ is the cost of deploying unit length of coastal protection boom around staging area i, and OC is the unit price of recovered oil. In the objective function, the first three terms are the cost of the cleanup operations, the fourth terms is the cost of coastal protection operations, and the last term is the credit resulting from the recovery of the emulsified oil.

3.4 Solution Approach

The resulting model is an MIDO, which can be solved by a number of approaches. [17, 18, 19, 20]. Because of the problem size and structure, in this work we use a simultaneous approach in order to integrate the continuous-time representation in

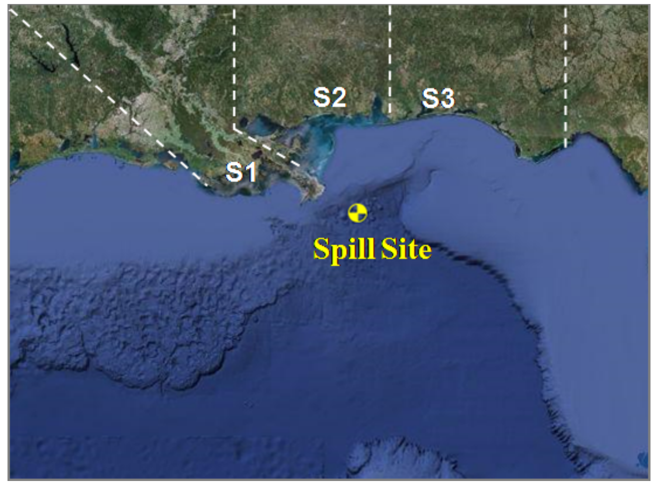

Figure 1: Oil spill site and locations of the three staging areas for the case study

the oil weathering model and the discrete time representation in the planning model. In the simultaneous approach, the MIDO model is fully discretized based on orthogonal collocation on finite elements and then is reformulated into an equivalent MINLP problem. First, the entire planning horizon is divided into a number of finite elements. Within each finite element an adequate number of internal collocation points is selected. Using several finite elements is useful to represent dynamic profiles with nonsmooth variations. Thus, the differential and algebraic variable profiles are approximated at each collocation point by using a family of interpolation polynomials. To integrate the continuous- and discrete-time representations, we consider one time period as a finite element in the discretization process. In this way, the index t represents not only the discrete time periods but also the finite elements, and the length of finite element t is the same as the length of the corresponding time period (H_t).

The reformulated MINLP, which is highly nonlinear nonconvex, requires careful initializations to avoid numerical difficulties. To obtain a "good" starting point, we use the MILP model given in [10, 21] for initialization. The approximate MILP model, which is obtained by decoupling the ODE from the discrete-time response planning model, implicitly considers the oil weathering process in the response planning by assuming the time trajectory of the slick thickness is not affected by response operations.

4. Example

Our case study is based on the *Deepwater Horizon BP* oil spill in the Gulf of Mexico. There are three major staging areas for the response operations: S1, S2, and S3. Their locations, along with the spill site, are given in the map in Figure 1. The minimum distances between the three staging areas and the oil spill site are 60 kilometers, 120 kilometers, and 180 kilometers, respectively. In this case, we assume the oil slick drifts toward the shore as a result of wind and current directions. The lengths of the booms required to protect the sensitive coastline near the three staging areas are 200 kilometers, 180 kilometers, and 300 kilometers, respectively. The spilled oil is considered as crude oil with an API degree of 25. The initial spill amount is 10,000 m^3, and the oil releases continue for 42 days with a constant rate of 10,000 m^3/day. The cleanup target is that no more than 1,500 m^3 of oil remain on the sea surface after the response. The cleanup facilities include three types of mechanical systems, two types of in situ burning systems, and three types of dispersant application systems (vessel, helicopter, and C-130). Each system has a corresponding operating capacity, available number, operational cost, and response time. All the other input data are available upon request.

All the computational studies were performed on an IBM T400 laptop with an Intel 2.53 GHz CPU and 2 GB RAM. DICOPT was used as the MINLP solver. The MILP problems were solved by using CPLEX 12.2 with an optimality tolerance of 10^{-9}. The nonlinear programming subproblems were solved with KNITRO41 with an optimality tolerance of 10^{-6}. We consider a response time span of 76 days in this example. The resulting MINLP problem includes 2,052 discrete variables, 11,482 continuous variables, and 14,006 constraints. We first solve the approximate MILP problem and use its solution as the starting point of solving the MINLP problem. The solution process takes a total of 139 CPU-seconds. We note that the problem becomes "infeasible", when we solved the MINLP directly without the initialization step.

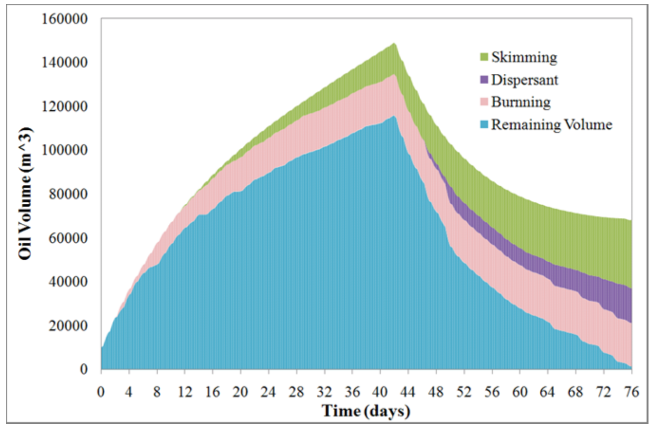

Figure 2: Time trajectories of the oil volumes removed by three methods and remaining on the sea surface

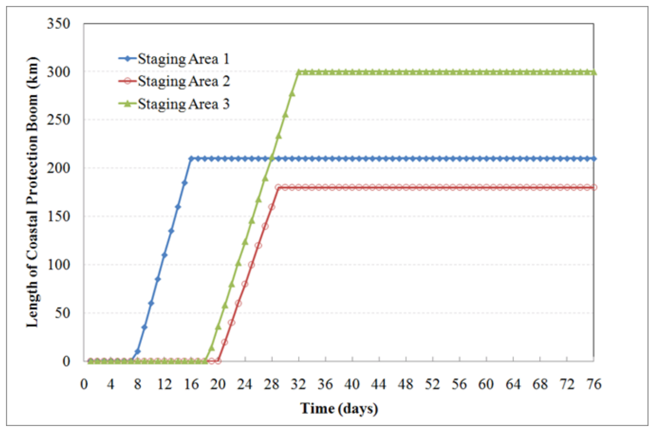

Figure 3: Optimal length of coastal protection boom

The results are given in Figures 2 and 3. Figures 2 show the time trajectories of the oil volume throughout the response operations when time span is 76 days. The drop lines are for the collocation points in the finite elements. We can see a trend from these figures that the volume of remaining oil first increases from Day 0 to Day 42 and then decreases, because the oil was being released at a constant rate to the sea surface before Day 42. These figures show that dispersant application is usually the most favorable cleanup method because of its flexibility in various weather conditions, although it might not be available in the early stage of the response. Skimming is also a major cleanup method, because the total amount of chemical dispersant used is constrained by regulation and mechanical cleanup can gain credit from oil recovery. Figure 3 shows the length of coastal protection booms deployed in the three staging areas when the response time span is 76 days. We can see that the three staging areas start to deploy booms from Day 8, Day 21, and Day 19, respectively. The different starting days are due to the different boom deployment rates and different locations of the staging areas. S1 deploys the booms first, because it has shortest distance to the oil spill site; Although S2 is closer to the spill site than S3, S3 starts to deploy the booms earlier than S2, because S3 requires much longer booms to protect the coast and longer deployment time.

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